

**PULSE DELTAMODULATION TECHNIQUES**

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**BRYAN COWAN**

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# PWM PULSE MODULATION DEL DELTA MODULATION

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PULSE DELTAMODULATION TECHNIQUES

by

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Lieutenant Colonel, United States Army

Submitted in partial fulfillment  
of the requirements  
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## PREFACE

The work described herein was done at the US Naval Postgraduate School during school year 1955 and by use of the facilities of the Pacific Aeronautic Library, Institute of Aeronautical Sciences, Los Angeles, California.

The equipment described and the analyses given in this work are intended to extend the scope of knowledge of the deltamodulation field and to stimulate interest in simpler encoding devices.

Credit is due Professors Robert E. Bauer, Earl G. Goddard and Robert L. Miller for advice and counsel during the preparation of this work.



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# TABLE OF SYMBOLS AND ABBREVIATIONS

$e_m$  - the intelligence signal input to a transmission system

$f_p$  - pulse frequency

$\tau$  - pulse amplitude

$f_s$  - a frequency component of the intelligence signal

$A$  - amplitude of a wave

$N$  - quantizing error voltage magnitude; also an integer number ( $n$ )

$\tau$  - pulse spacing or pulse repetition time

$\alpha$  - pulse width

$\omega$  - angular frequency

$C_n$  - amplitude of a harmonic component



# CHAPTER I

## INTRODUCTION

Communication by modulating a pulse carrier with the intelligence signal was a natural outgrowth of the "shrinking" world. Communication was required to keep pace with the faster and faster transportation means. Lengthening of communication lines involved many intelligence signal regeneration processes, and demanded that the noise and distortion inevitably present not be cumulative in these processes. Pulse modulation, particularly pulse code modulation, satisfies this requirement.

Pulse modulation techniques, having been described in general terms years ago--indeed as early as wire telegraphy--took a great forward step during World War II. Hurried study and experimentation in the radar field and with microwave techniques gave widespread practical application importance to pulse modulation systems.

A comprehensive theory of communication has been recorded during and following World War II, due largely to the work of Norbert Wiener, Claude Shannon, B. M. Oliver, J. R. Pierce, W. G. Tuller and others. This theory provides analytical explanations for the capabilities and limitations of the various modulation techniques employed in the presence of noise.

Interest in the pulse modulation techniques has been very evident both in the United States and abroad during the past decade. The several forms of pulse modulation have been studied and applied to practical





communication requirements. This paper deals with delta modulation as a special case of pulse code modulation. The delta modulation technique was advanced by L. J. Libois [7] in 1951 and has received attention in Holland [5] and Austria [8].

Systems described in current literature are either of the feed-back or the pulse frequency synthesis type. Some disadvantages accrue to each of these methods of producing delta modulation. The feedback system was the first to be used, and when used with widely varying amplitudes of input signal, such as speech, tends toward amplitude saturation and also suffers from stability problems. These problems have been described in other work [4]. The pulse frequency synthesis method of delta modulation encoding has been shown to materially reduce the magnitude of the difficulties cited above. This technique, however, involves use of a special blocking oscillator transformer with three windings and of rather complex design, construction and adjustment.

In addition to the two techniques of encoding for delta modulation mentioned above, a third technique is considered. Here series differentiation of a frequency limited intelligence signal is performed and the resulting waveform is shaped and used to determine the polarity of a pulse generator output so that

- a. when  $\frac{dem}{dt} = 0$ , output pulse train consists of alternately positive and negative pulses,
- b. when  $\frac{dem}{dt} > 0$ , output pulse train is positive, or
- c. when  $\frac{dem}{dt} < 0$ , output pulse train is negative.

The purpose of this paper is to record some work done toward employment of the delta modulation techniques to a transmission path where





fidelity is not a consideration, but rather where the signal-to-noise advantage of pulse code modulation technique is demanded for satisfactory communication.

Pulse deltamodulation purports to provide the signal-to-noise and regeneration advantages of conventional pulse code systems with but very simple terminal equipment. Brief consideration of pulse modulation principles will enhance appreciation of the deltamodulation techniques.



## CHAPTER II

### PRINCIPLES OF PULSE MODULATION

The techniques of transmission of intelligence by causing it to vary some detectable characteristic or property of a pulse train are embraced by pulse modulation. Of interest to this study are the degrees of redundancy exhibited by the various techniques. Descriptions of modern forms of pulse modulation appear in the literature and various texts, and are not repeated herein. For purposes of reference the following specific forms of pulse modulation are in modern use:

Pulse Amplitude Modulation (PAM)

Pulse Duration Modulation (PDM)

Pulse Frequency Modulation (PFM)

Pulse Time Modulation (PTM)

Pulse Code Modulation (PCM)

Pulse Deltamodulation

In the PAM, PDM, PFM and PTM, the fundamental modulation process is analog in that a specific characteristic of the pulse train is caused to assume exactly as many states as are used to describe the modulating time function.

In pulse code modulation and deltamodulation, the modulation process is fundamentally digital in that the number of states assumed by the modulated function is many times less than the number of states used to describe the modulating time function.



Every technique except deltamodulation possesses a high order of redundancy in that complete modulating function amplitude information is included in the information derived from each received pulse (or pulse group in case of PCM). In deltamodulation, the information gleaned from each pulse provides only incremental information relative to the amplitude of the modulating time function. It is obvious therefore that information storage must be accomplished in the system to permit reconstruction at the receiver of a transmitted signal. Normally information storage is accomplished in the receiver by an integrator, as will be fully discussed later. This integration of incremental information leads to possible cumulative error in event of pulse loss due to noise. In conventional PCM, however, due to the redundancy, loss of one or more pulses does not make for such a cumulative error.

In some senses, the deltamodulation technique has been expressed as a special case of an  $n$ -digit PCM system where  $n$  is unity. Following this hypothesis leads to the impossible requirement that instantaneous values of the time function amplitude be expressible in a uni-digit code. L. J. Libois, in his articles on deltamodulation, at this point justifies the name and represents incremental elements of the time function by the uni-digit code. This results in the transmission of information only as to the polarity of the rate of change of the time function. The quantization of the signal is therefore a quantization in time only.

For the purposes of this paper, since the term is not rigorously defined elsewhere, deltamodulation is defined as modulation of a pulse carrier in accordance with the rate of change of the modulating function.





Thus defined, the process is not a discrete form of pulse modulation, but can involve any of the accepted techniques such as PAM, PFM, etc, but must be based upon incremental changes in the modulating signal rather than the amplitude of the modulating signal.

As previously indicated, two techniques for generating dltamodulation have been described in the literature. A third technique follows from the postulated definition of dltamodulation. The three are:

- (1) Feed-back encoding
- (2) Frequency synthesis encoding
- (3) Encoding by gating

In order to make comparisons of these techniques some descriptive information will be repeated from other references.

The feed-back encoding technique involves amplitude sampling (at a fixed rate) of the modulating function, comparison of this amplitude with the integrated output and generation of a positive pulse if the sampled amplitude is greater, or a negative pulse if the sampled amplitude is smaller. A block diagram of a possible system employing the feed-back technique is shown in Figure 1. The decoding device needs only to integrate and effect low-pass filtering as shown in Figure 2. Waveforms arising in such a system are shown in Figure 3. The modulating function shown is a complex waveform as in (a). The pulse train to be modulated is shown in (b), to emphasize that both frequency and amplitude are fixed. The decoder used as the feed-back path may consist of a simple integrator and generates the waveform shown in (c), which is fed into the comparison circuitry of the encoder. The encoded output (d)





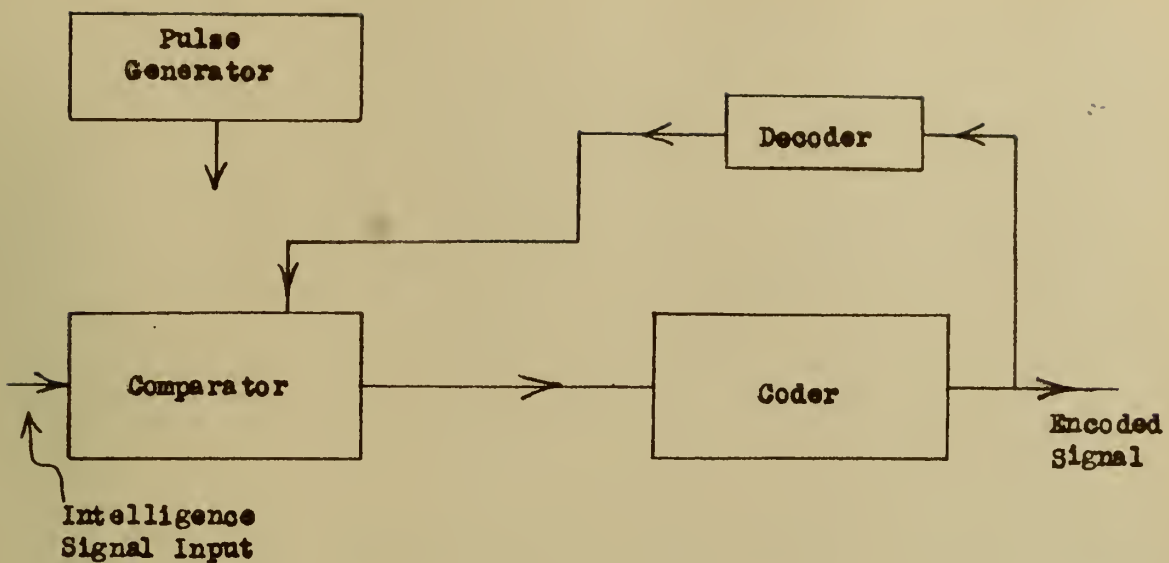
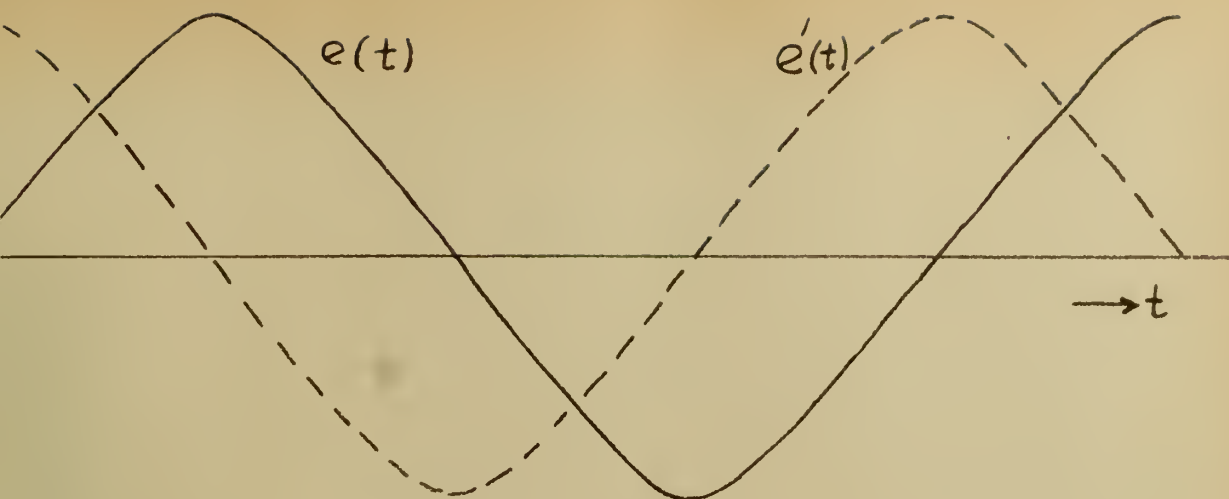


FIGURE 1. Feedback Type Encoder



FIGURE 2. Deltamodulation decoder

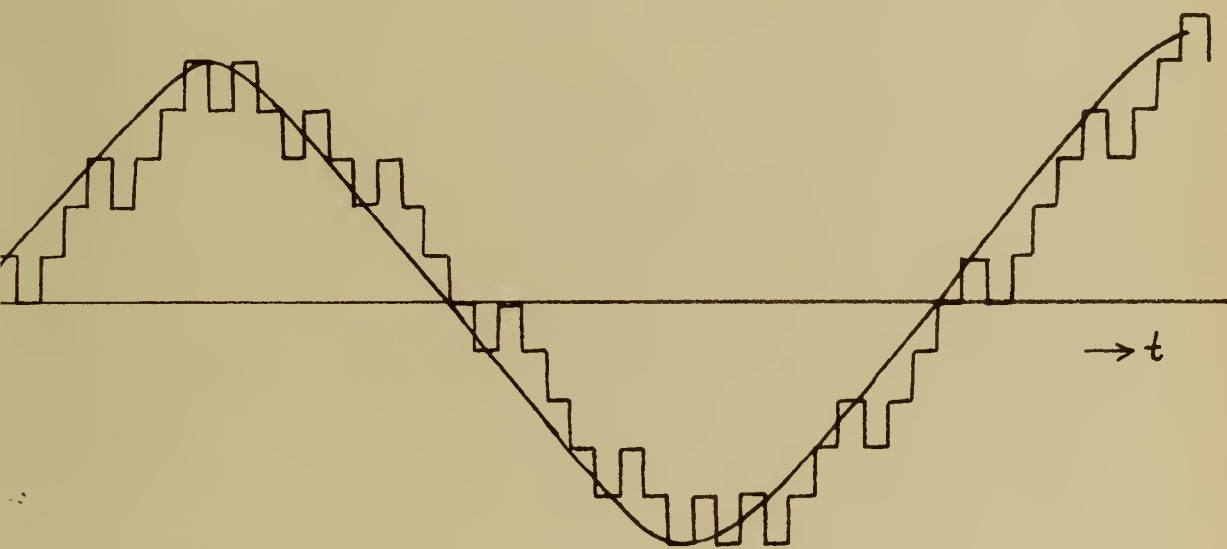




(a)



(b)



(c)



(d)

FIGURE 3. Feedback Encoding Waveforms





FIGURE 4. Decoder Waveforms (For feedback encoded input).



indicates that if, at the sampling instant, the modulating amplitude is greater, a positive output pulse occurs next, whereas if the decoder output is greater, a negative output pulse occurs next. It is intuitively apparent that at the receiver when the pulse train is integrated and low-pass filtering accomplished, the waveforms will be as shown in Figure 4.

This is historically the first technique used for generation of delta modulation [ 7 ].

It should be pointed out that dual polarity pulses such as shown in Figure 3(d) need not be transmitted where radio link is the transmission means. If either the positive or negative pulse train is clipped and the other transmitted by the radio carrier, for example, the absence of a pulse at the receiver can readily be interpreted as the existence of a negative pulse at the transmitter. Of course, in the presence of high noise levels, pulse regeneration will be required prior to decoding at the receiver. These comments apply to all the delta-modulation techniques.

The second general technique for obtaining delta modulation is that of pulse frequency synthesis. This technique stemmed from the saturation and stability problems which plague the feed-back method. No feedback of information is required. The method is shown in block diagram in Figure 5. Waveforms which apply to this system are shown in Figure 6. The modulating function (a) is differentiated resulting in (b). The instantaneous values of voltage (b) are used to control the variable frequency pulse generator so that its output is as shown in (c). Timing pulses (d) and the variable frequency pulses (c) are





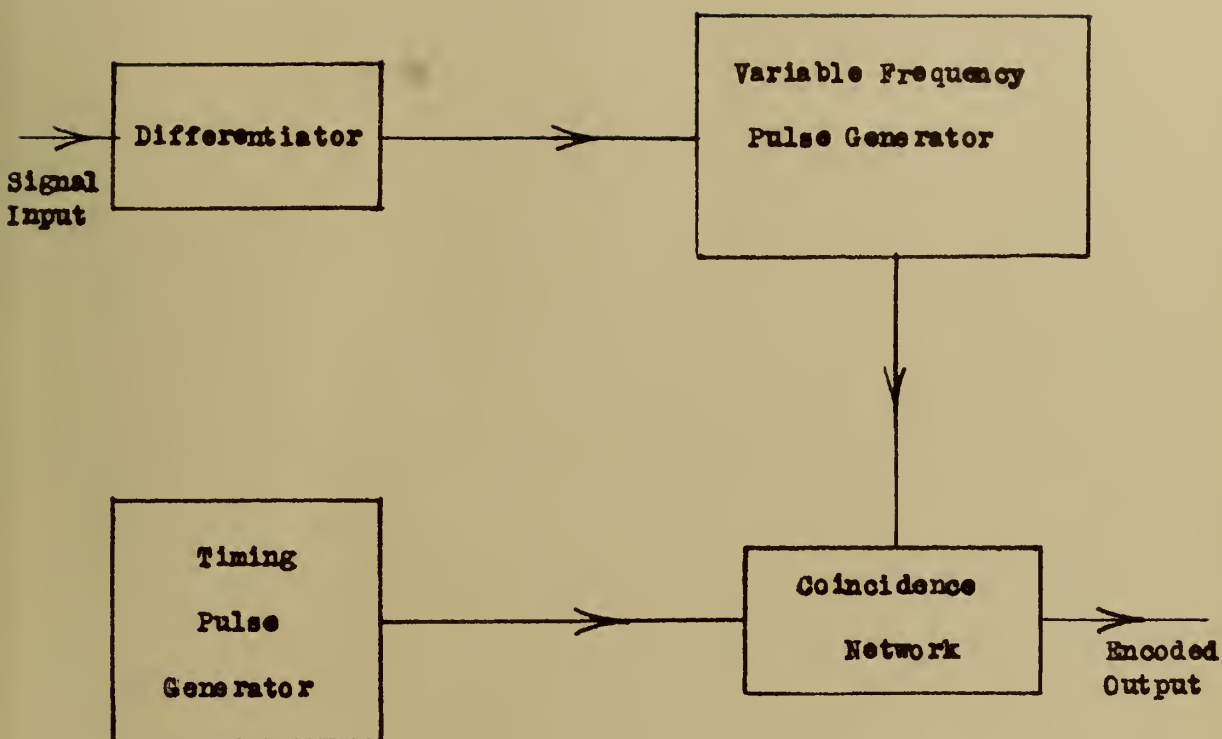


FIGURE 5. Pulse Frequency Synthesis Encoder



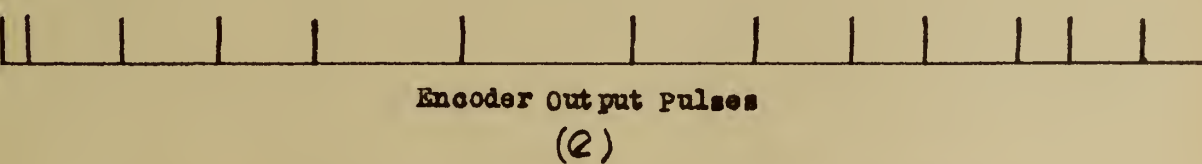
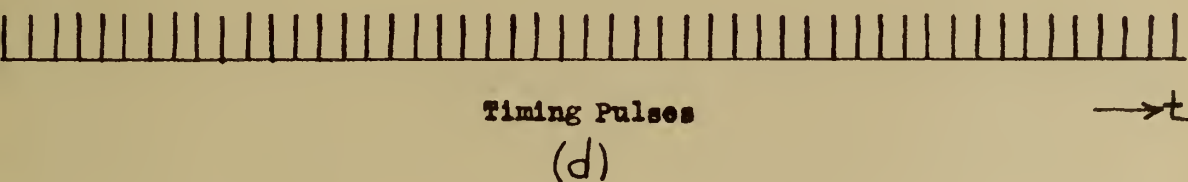
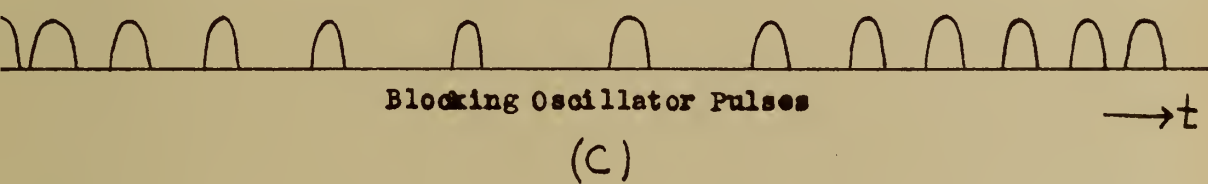
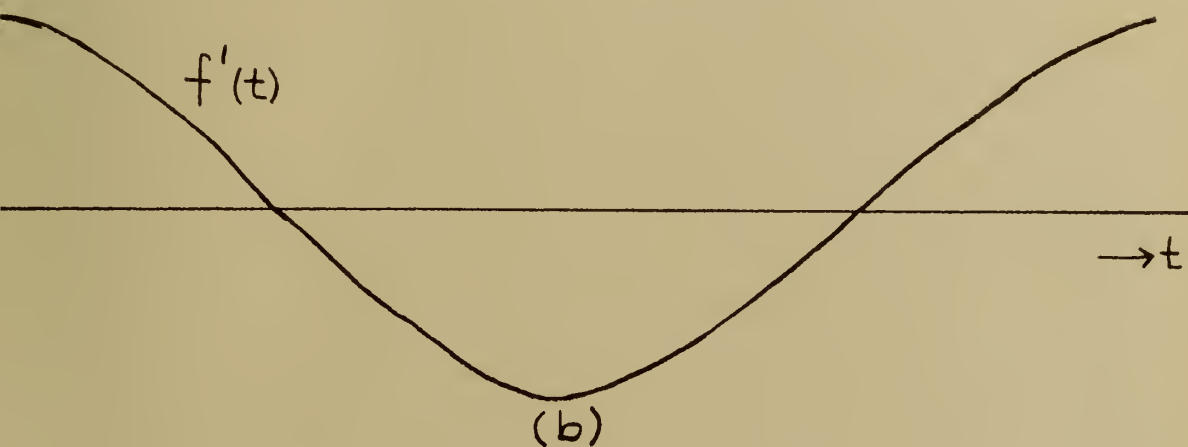
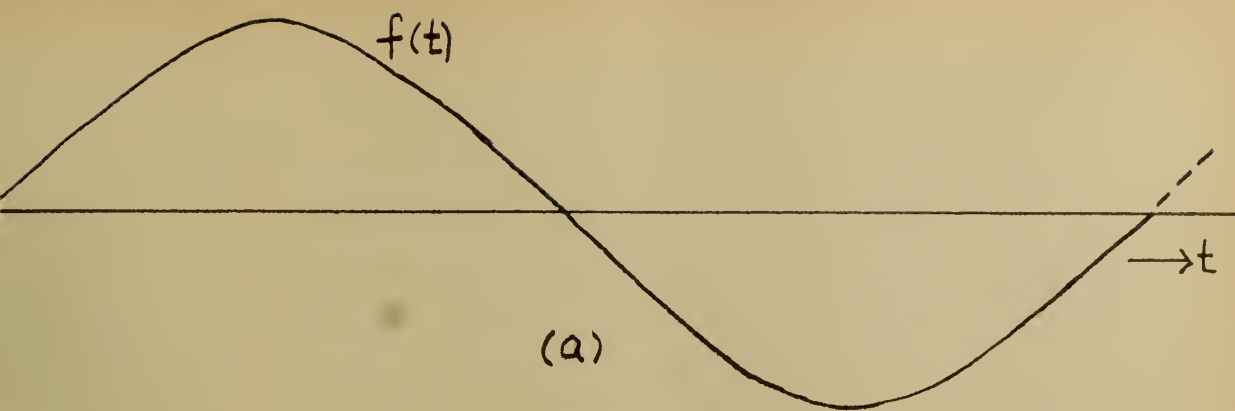


FIGURE 6. Pulse Frequency Synthesis Encoder Waveforms



introduced in a coincidence network so that only those timing pulses which are immediately preceded by a pulse from the variable frequency pulse generator appear in the output (e). Decoding of this type of output is accomplished in the same manner as shown in Figure 4 for the feed-back system.

The output of the pulse frequency synthesis encoder has been shown to be exactly equivalent to the output of the feedback encoder [ 6 ], while the former possesses far superior saturation and stability characteristics. Two criteria must however be met:

- (1) The timing pulse frequency must be at least twice the mean frequency of the variable pulse frequency generator
- (2) The frequency of the variable pulse frequency generator must vary essentially between zero and the timing pulse frequency as the rate of change of the modulating function varies from its maximum negative value to its maximum positive value.

The third method of producing a deltamodulation signal has been termed for the purpose of this paper "the gating encoder". This method of encoding may be accomplished as shown in block diagram form in Figure 7. The waveforms applying are shown in Figure 8. In this system, a modulating signal (a) is differentiated, amplified and clipped to give the gating voltage (b) which is used to modulate a pulse train (c) so as to give a pulse output (d). Decoding of this waveform by the conventional integrator-low-pass filter decoder is shown in Figure 8(e). The technique is simply to transmit positive pulses when the rate of change of the modulating voltage is positive and negative pulses when



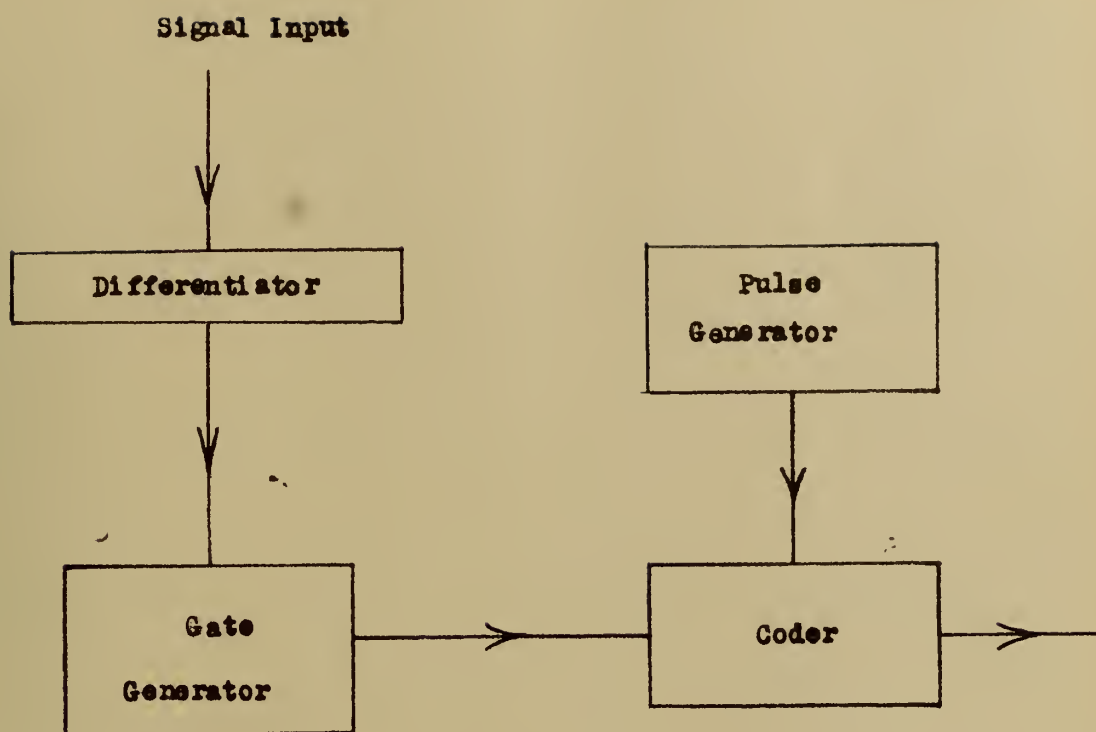


FIGURE 7. Gating Encoder.





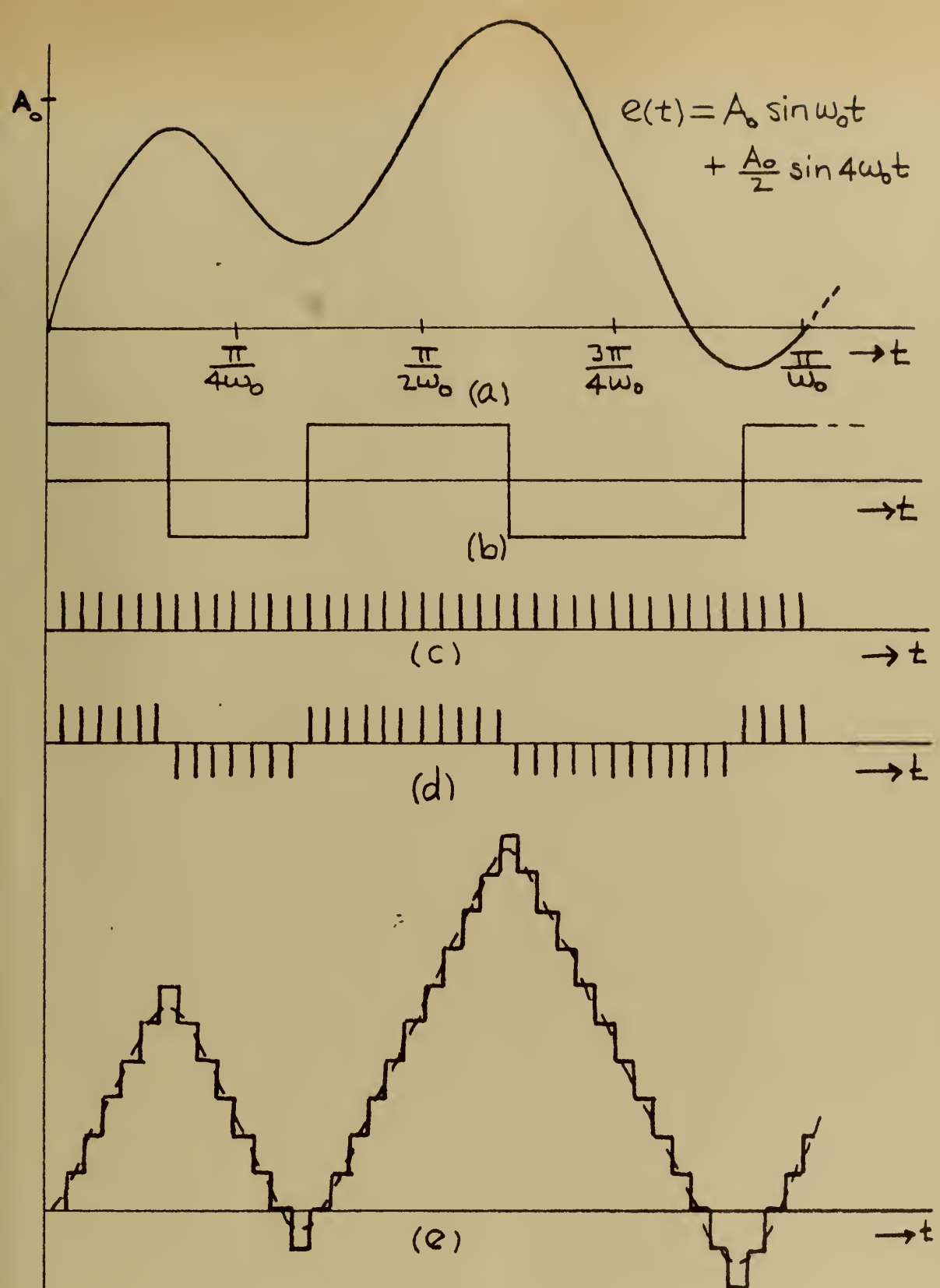


FIGURE 8. Waveforms of Deltamodulation Gating Encoder



the rate of change of the modulating voltage is negative.

The definition of dltamodulation given on page 5 clearly allows all three of the techniques just described. The definition postulated however does not limit itself to these three, but could apply to any system in which the modulating signal rate of change is used in the sense of removing the redundancy of the transmitted information. Some analyses of the transmitted information in each of the three systems are given in Chapter III.



## CHAPTER III

### ANALYSIS OF PULSE DELTAMODULATION TECHNIQUES

Three methods for generating a deltamodulation waveform, as defined on page 5 herein, have been given in Chapter II. In approaching the subject of deltamodulation analysis, it is instructive to consider the information contained in the encoded signals generated by each of the three methods.

The feedback encoding technique involves information storage in the sense that a decoder or "stepped image generator" (after W. F. Armstrong [1]) is employed in the encoder. By reference to the waveforms shown on page 8, it can be seen that presence of the output of a pulse of specified polarity at a sampling instant depends not solely on the rate of change of the input signal with respect to time, but rather upon the difference between the value of the input signal and the decoded waveform. Then the rate of change of the input signal can be positive, but because of the value of the stepped image, a negative pulse may appear in the output. Thus the encoded waveform obtained by the feedback process contains more information than merely the polarity of the rate of change of the input signal. Taken as a complete pulse train, it is apparent that the positive and negative pulse sequences in the encoded waveform give both polarity and magnitude information about the rate of change of the input signal. This will be more apparent when the distortions arising from the deltamodulation processes are considered.

The pulse frequency synthesis encoding technique as depicted by





waveforms on page 12 may be considered from the same standpoint--what information about the input signal is contained in the encoded output? The presence of the density modulator and control of it by a voltage proportional to the rate of change of the input signal furnishes a ready answer. The presence of the pulse means positive rate of change of input signal while absence of the pulse stems from a negative input signal rate of change. But further, the encoded signal contains information as to the magnitude of the input signal rate of change, by virtue of the pulse frequency variations, even though these may be quantized in time by the timing pulse generator.

Thus the encoded outputs of both the feedback and the pulse frequency synthesis encoders contain amplitude and polarity information about the input signal rate of change. The question arises as to the consequences of demanding that the encoded signal contain only information as to the polarity of the rate of change of the input signal. This is essentially the third or gating encoder. Note in the waveforms shown in figure 8 that no information storage is involved in the encoding process. Only the fact of positive or negative input signal derivative determines the polarity of the pulse output, and no information as to magnitude of that derivative is used. It is apparent here that the three techniques of encoding being considered do not constitute all the possibilities. It is not necessary (merely convenient!) to vary frequency in accordance with the amplitude of the input signal rate of change; rather pulse amplitude or any of the other pulse modulation techniques may be used if proper circuit timing design retains the rate of change of





polarity information in the encoded output.

The analysis of the encoding techniques is approached by determining the distortions which cause the decoded waveform at the receiver (decoder) to differ from the signal input waveform at the encoder. Only system distortion is considered, which may take the forms of signal amplitude distortion, quantizing distortion or harmonic distortion. The latter two are not independent manifestations, but are customarily obtained by different techniques.

Signal amplitude distortion arises only if a determinable amplitude limitation is exceeded by the input signal or its rate of change. It has been characteristically termed "diagonal distortion". Armstrong has shown [1] that in a feedback encoder, for an input wave of the form  $A \sin \omega t$ , the slope is  $\omega A \cos \omega t$  which has maximum absolute values  $\omega A$  at  $\omega t = n\pi$ , where  $n$  can be zero or any integral value:

$$A_{max} = \frac{f_p \sigma}{2\pi f_s}$$

where  $f_p$  is the sampling rate

$\sigma$  is the height of one step in the stepped image

$f_s$  is the input signal frequency

if no diagonal distortion is to occur. This expression may be written in the form

$$\frac{A_{max}}{\sigma} = \frac{f_p}{2\pi f_s} \quad (3-1)$$

which shows that if  $f_s$ , the signal frequency, increases, then  $A_{max}$  must decrease if  $\sigma$  and  $f_p$  are to remain constant.



It has been shown that for the purposes of distortion analyses, the technique of producing a dltamodulation signal by the pulse frequency synthesis technique is equivalent in all respects to the feedback encoding technique [ 6 ].

Observation of the waveforms shown on page 15 for the gating encoder will show that the same maximum slope relationship applies to this technique as was developed for the feedback encoding process, so again equation (3-1) applies.

It therefore appears that a limiting frequency (amplitude) exists for all three of the types of encoding being considered, and that the relationship between the variables of the system and this limiting frequency is given by equation (3-1).

Quantization-generated distortion for feedback (and pulse frequency synthesis) encoders has been analyzed by deJager [ 5 ]. An approximation is developed based on a stepped image system performing single integration and the assumption of irregular non-periodic error voltage of continuous frequency spectrum existing. Lacking periodicity, no correlation is assumed to exist for time intervals much greater than  $\frac{1}{f_p}$ , since this is the order of the time function which will be passed by the receiver low-pass filter. The following proportionality is obtained:

$$N^2 \propto \frac{f_s}{f_p} \sigma^2 \quad (3-2)$$

which may be written in terms of voltage as

$$N = k \sqrt{\frac{f_s}{f_p}} \sigma \quad (3-3)$$

where N is the rms quantizing error voltage

k is a constant of proportionality



and  $f_s$ ,  $\nabla$  and  $f_p$  have the same meanings as used previously.

Equation (3-3) shows that the rms error voltage increases with increase in pulse height or increase in input signal frequency.

The gating encoder waveforms shown in Figure 8 do not clearly indicate quantizing distortion content present in the encoded output. However, consistent with the analyses of feedback and pulse frequency synthesis techniques, it can be assumed that the distortion in the gating encoder definable as quantizing distortion to the exclusion of harmonic distortion, arises from the same characteristic type of triangular error voltage. So the same order of quantizing distortion as given by equation (3-3) applies to the gating encoder. It should be noted, however, that this is a distortion fundamentally related only to the quanta of the encoding process.

The analytic determination of harmonic distortion in any of the deltamodulation encoding processes has not, as far as the author can ascertain, been rigorously accomplished from a mathematical standpoint. The non-linearity of the processes involved makes for quite complex manipulations, and has led most investigators to content themselves with data obtained by use of spectrum analyzers to measure the characteristics of a physical system. For the feed-back type encoder, Armstrong [1] concludes, based on observed characteristics, that harmonic distortion is given approximately by

$$D = \frac{k}{f_p} \quad (3-4)$$

where  $D$  is harmonic distortion in per cent

$k$  is a constant associated with a particular system (of order about





$$3 \times 10^5)$$

$f_p$  is the pulse frequency.

Of some related interest is the work of the Laboratoire Central de Telecommunications, Paris, France [6] which leads to the definition of an "equivalent number of pulse-code modulation levels" for a given deltamodulation system. The work indicates that the number of PCM quanta employed in the equivalent system is

$$N = \frac{1}{4\pi} \left( \frac{6f_p}{f_s} \right)^{\frac{3}{2}} \quad (3-5)$$

Analysis of the process of encoding by employing gate circuits has been undertaken in the course of preparation of this paper. The approach used was to consider the signal input to be a simple sine wave of amplitude unity (since gain control allows arbitrary choice) and frequency  $\omega_0$ , as shown in Figure 9(a), page 23. This input,  $e(t)$  and its derivative  $\frac{1}{\omega_0} e'(t)$  are shown in time relation to the other waveforms involved in the encoding process. The pulse train  $P(t)$  is shown in (b) while the encoded pulse output, termed  $f(t)$ , is shown in (d). The gating voltage waveform is shown in (c). It will be instructive to determine the expression for the time function represented by the output of the decoder after integration and low-pass filtering of the encoded pulse function. If the function which results from integration of the encoded pulse train (Figure 9(d)), say  $h(t)$  as shown in Figure 9(e), can be expanded in a Fourier series then low-pass filtering can be interpreted as reducing to zero all the coefficients of terms involving frequencies above the filter cut-off, or  $\omega > 2\pi f_{\text{cut-off}}$ . Thus the remaining





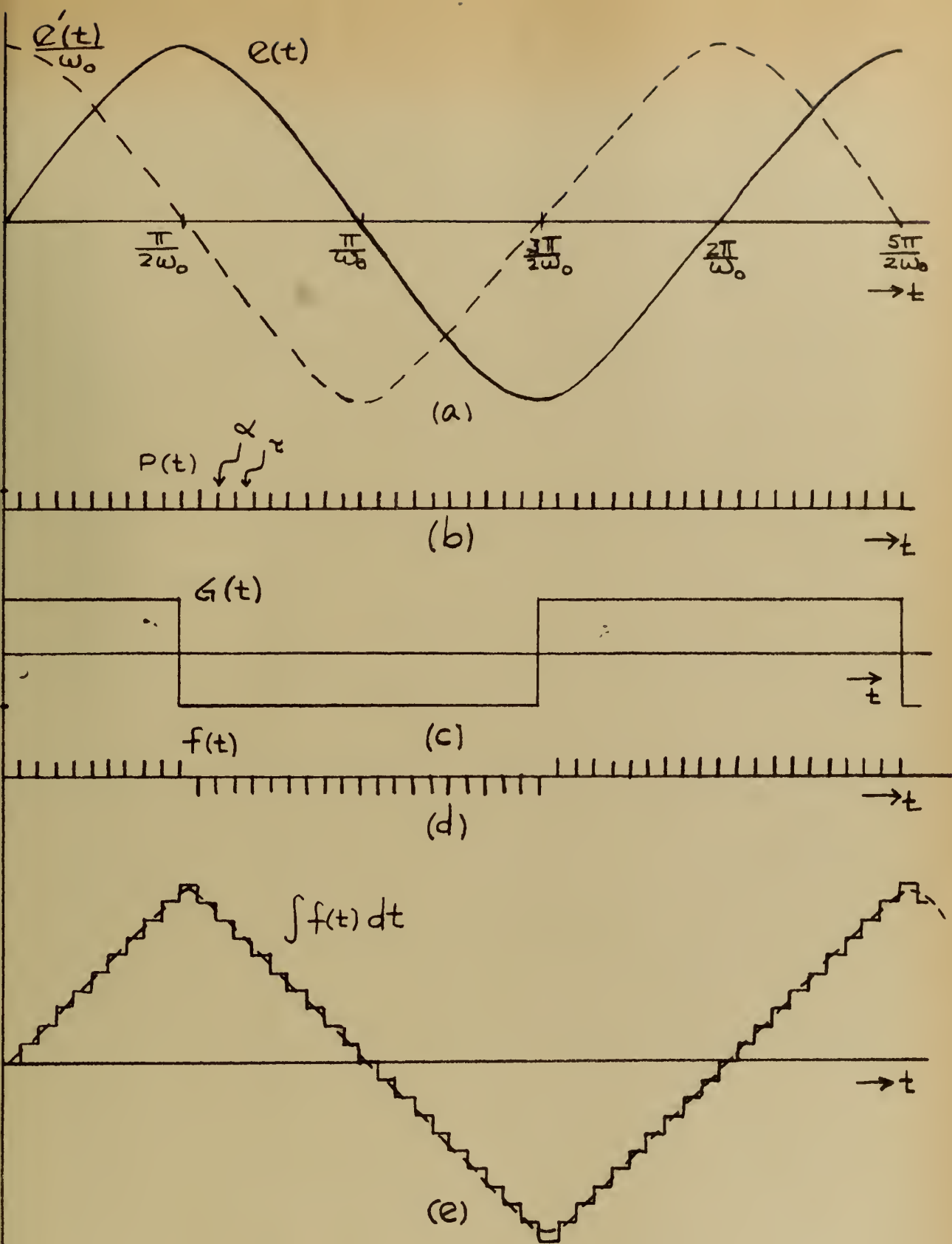


FIGURE 9. Gating Encoder Waveforms of Sine Wave Input



terms in the  $h(t)$  series expansion give the intelligence and distortion components of the output waveform.

The encoded waveform  $f(t)$  can be generally expressed in the time domain as

$$f(t) = P(t) \cdot G(t) \quad (3-6)$$

in which  $P(t)$  describes the pulse sequence (pulses of width  $\alpha$  and spacing  $\tau$ ) so that

$$P(t) = A \sum_{n=0}^{\infty} [u(t - n\tau) - u(t - n\tau - \alpha)] \quad (3-7)$$

and  $G(t)$  describes the gating (modulating function), so that for the assumed sine wave input,

$$G(t) = u(t) - \sum_{m=0}^{\infty} (-1)^m 2 u(t - \frac{\pi}{2\omega_0} - m \frac{\pi}{\omega_0}) \quad (3-8)$$

Substituting (3-8) and (3-7) in (3-6) gives

$$\begin{aligned} f(t) &= A \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [u(t - n\tau) - u(t - n\tau - \alpha)] \left[ u(t) \right. \\ &\quad \left. - (-1)^m 2 u(t - \frac{\pi}{2\omega_0} - m \frac{\pi}{\omega_0}) \right] \\ f(t) &= A \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [u(t) u(t - n\tau) - u(t) u(t - n\tau - \alpha) \\ &\quad - u(t - n\tau) (-1)^m 2 u(t - \frac{\pi}{2\omega_0} - m \frac{\pi}{\omega_0}) \\ &\quad + u(t - n\tau - \alpha) (-1)^m 2 u(t - \frac{\pi}{2\omega_0} - m \frac{\pi}{\omega_0})] \quad (3-9) \end{aligned}$$



So that the expression in the time domain for the encoded pulse function  $f(t)$  is made up of an infinite series of terms each of which has the general form

$$u(t-a) u(t-b)$$

Since the integration of the expression for the encoded pulse train appears simplest of accomplishment in the frequency domain, the theorem of real multiplication [3]

$$\mathcal{L}[f_1(t) f_2(t)] = \frac{1}{2\pi j} \int_{c_2 - j\infty}^{c_2 + j\infty} F_1(s-w) F_2(w) dw$$

may be applied by taking

$$f_1 = u(t-a) \quad \text{whence} \quad F_1(s-w) = \frac{1}{s-w} e^{-a(s-w)}$$

$$f_2 = u(t-b) \quad \text{whence} \quad F_2(w) = \frac{1}{w} e^{-bw}$$

so that

$$\mathcal{L}[u(t-a) u(t-b)] = \frac{e^{-as}}{2\pi j} \int_{c_2 - j\infty}^{c_2 + j\infty} \frac{1}{w(s-w)} e^{-(b+a)w} dw$$

Applying the partial fraction technique and letting  $a+b=c$ ,

$$\mathcal{L}[u(t-a) u(t-b)] = \frac{e^{-as}}{2\pi j s} \int_{c_2 - j\infty}^{c_2 + j\infty} \left[ \frac{e^{-cw}}{w} + \frac{e^{-cw}}{s-w} \right] dw$$



which may be rewritten

$$\int [u(t-a)u(t-b)] = \frac{e^{-as}}{2\pi sj} \left[ \int_{c_2-j\infty}^{c_2+j\infty} \frac{e^{-c\omega}}{\omega} d\omega - e^{-cs} \int_{s-c_2+j\infty}^{s-c_2-j\infty} \frac{e^{cx}}{x} dx \right]$$

where the second integral is obtained by letting  $x = s - \omega$ .

Thus it is seen that each of the integrals is of the form

$$\int \frac{e^{cx}}{x} dx$$

which does not evaluate to a closed expression. If the real multiplication process as above could have been performed, the result would have been a function in the frequency domain, say  $F(s)$ , involving the system constants  $A$ ,  $\alpha$ ,  $\tau$  and  $\omega_0$ . So integration of this function would have been accomplished by effecting the product of  $F(s)$  and  $\frac{1}{s}$  and finally inverse transformation to the time domain would have resulted in an infinite series of terms. Low-pass filtering could be accomplished by elimination of all terms involving frequencies in excess of the cut-off frequency of the low-pass filter. Harmonic distortion would then be given in terms of the system constants as above.

Although the detailed analysis outlined above is not accomplished, some information can be deduced. First, the integration process assumed to be performed in an ideal manner will result in distortion of the amplitudes of the various frequency components of the input signal. That is, if a high and a low voice frequency signal of equal amplitudes are introduced into the system, it is apparent that the amplitudes of the outputs will not be equal, and further that the lower frequency





input signal will have the greater output signal amplitude. And from this argument, it is seen that distortion in the amplitude of frequency components of the input signal will be proportional to  $\frac{1}{f_s}$ . Compensation

in an amplifier stage following the decoder can correct this distortion, by having a gain proportional to  $f_s$ . However the waveform output of the ideal integrator will be approximately a triangular configuration, as depicted in Figure 9(e). Assuming the output to be a perfectly triangular wave and to have arbitrary amplitude A, gives harmonic magnitudes which can be determined from (see Appendix I)

$$C_n = \frac{8A}{\pi^2 n^2} \quad n = 1, 3, 5, 7 \dots$$

and since the frequency range 500-3500 cps is concerned, harmonics as high as the seventh will be passed by the low-pass filter, so that the maximum distortion content will be of the order

$$D = \sqrt{\frac{\frac{C_3^2}{2} + \frac{C_5^2}{2} + \frac{C_7^2}{2}}{\frac{C_1^2}{2}}} \times 100$$

$$D \doteq 11.7 \%$$

in which  $C_3$ ,  $C_5$  and  $C_7$  are harmonic voltage amplitudes and D is the distortion per cent.

From the foregoing discussion of ideal integration and restoration of the amplitude relationships by compensation techniques, the output distortion content appears reasonable and indicates that complete intelligibility might be expected.



## CHAPTER IV

### ENCODING SYSTEMS

Functioning of practical systems employing the feedback and the pulse frequency technique has been covered in the literature [1,2,4,5,6,7]. For the purpose of this work, only brief descriptions of these systems are included in order to permit comparison with the gating encoder.

The feedback type of encoder is discussed by Robert Irving [4] and the saturation and stability characteristics of this type of system are evaluated. The interest in this technique for the most part now centers around development of improved feedback circuitry. The original circuit used for feedback was the simple integrator shown in Figure 10. In an effort to reduce noise and the threshold effects of the system, the double integrator circuit of Figure 11 was used. Threshold effect here means the difficulty which arises for existence of very small difference voltages between the sampled input and the stepped image integrator output. The functioning of these circuits in the system can be readily visualized by reference to the feedback system block diagram shown in Figure 1.

The pulse frequency synthesis technique appears to have substantially supplanted the feedback technique in the practical realm. A practical circuit of the pulse frequency synthesis type is shown in simplified version in Figure 12. In this circuit,  $V_1$  is the blocking oscillator which acts as a variable frequency pulse generator.  $C_1$  and  $R_1$  constitute a differentiating circuit such that the grid voltage of  $V_2$  is proportional



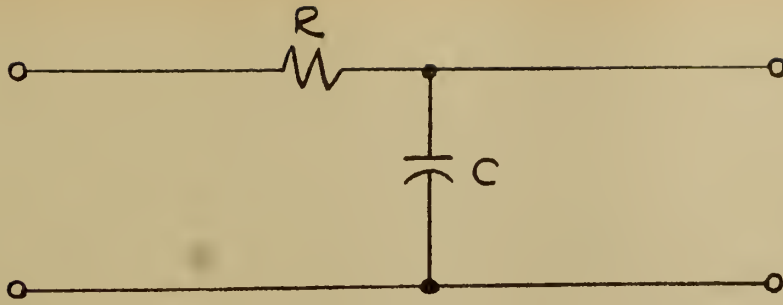


FIGURE 10. Simple Integrator Feedback Loop

$$RC \gg \frac{1}{f_p}$$

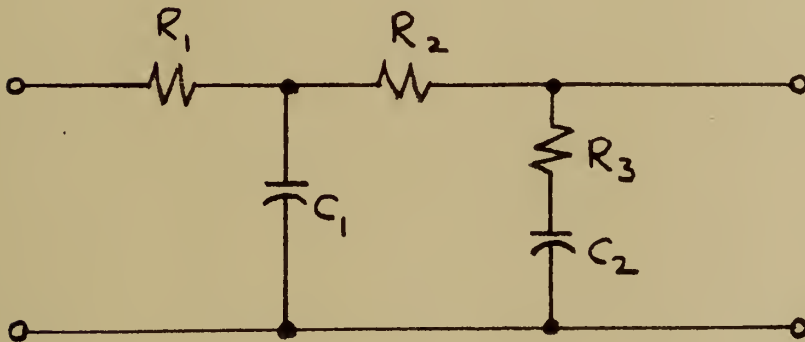


FIGURE 11. Double Integrator Feedback Loop

$$\frac{1}{f_p} \ll R_1 C_1 \text{ and } (R_2 + R_3) C_2$$



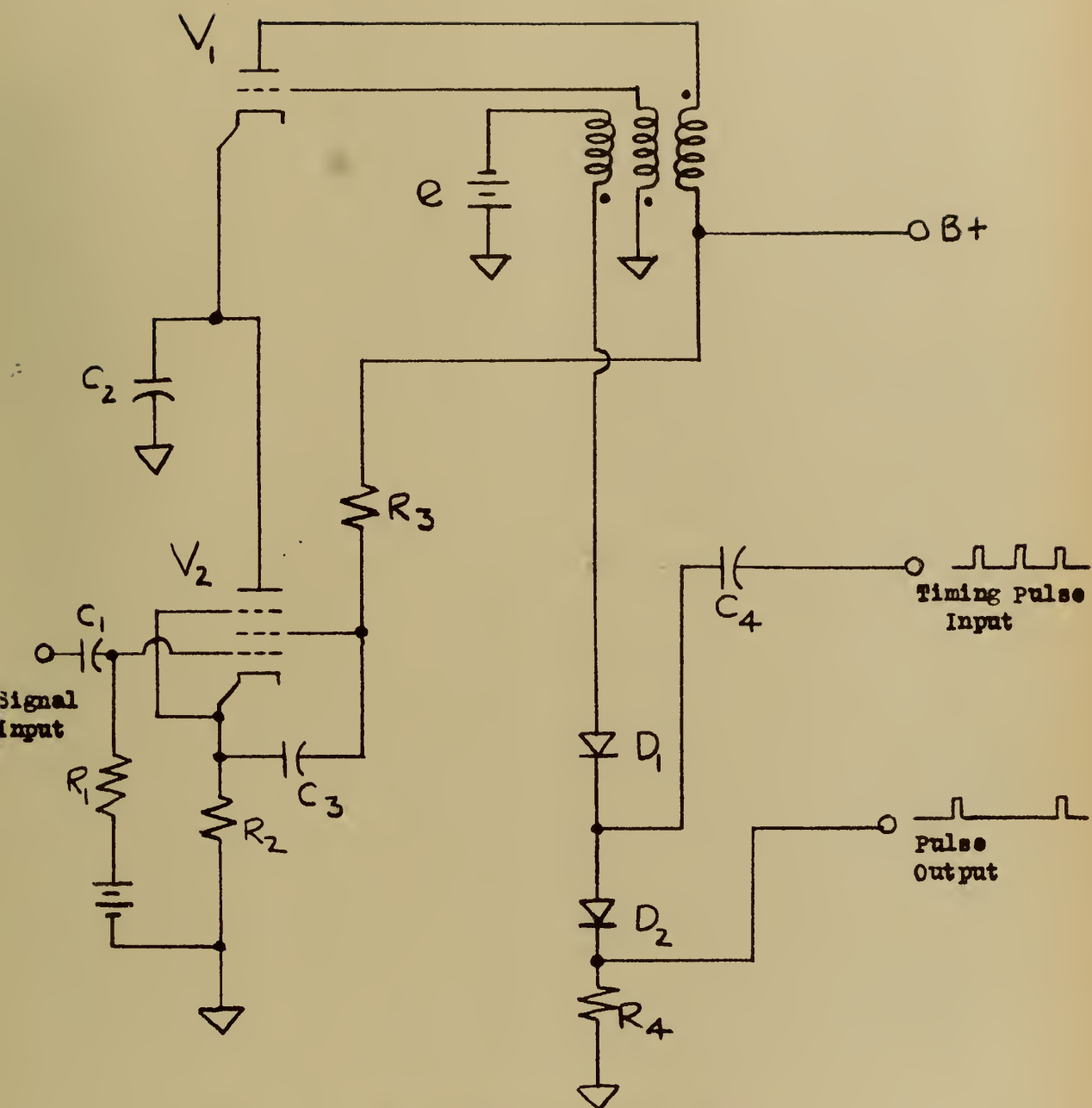


FIGURE 12. Pulse Frequency Synthesis Encoder.





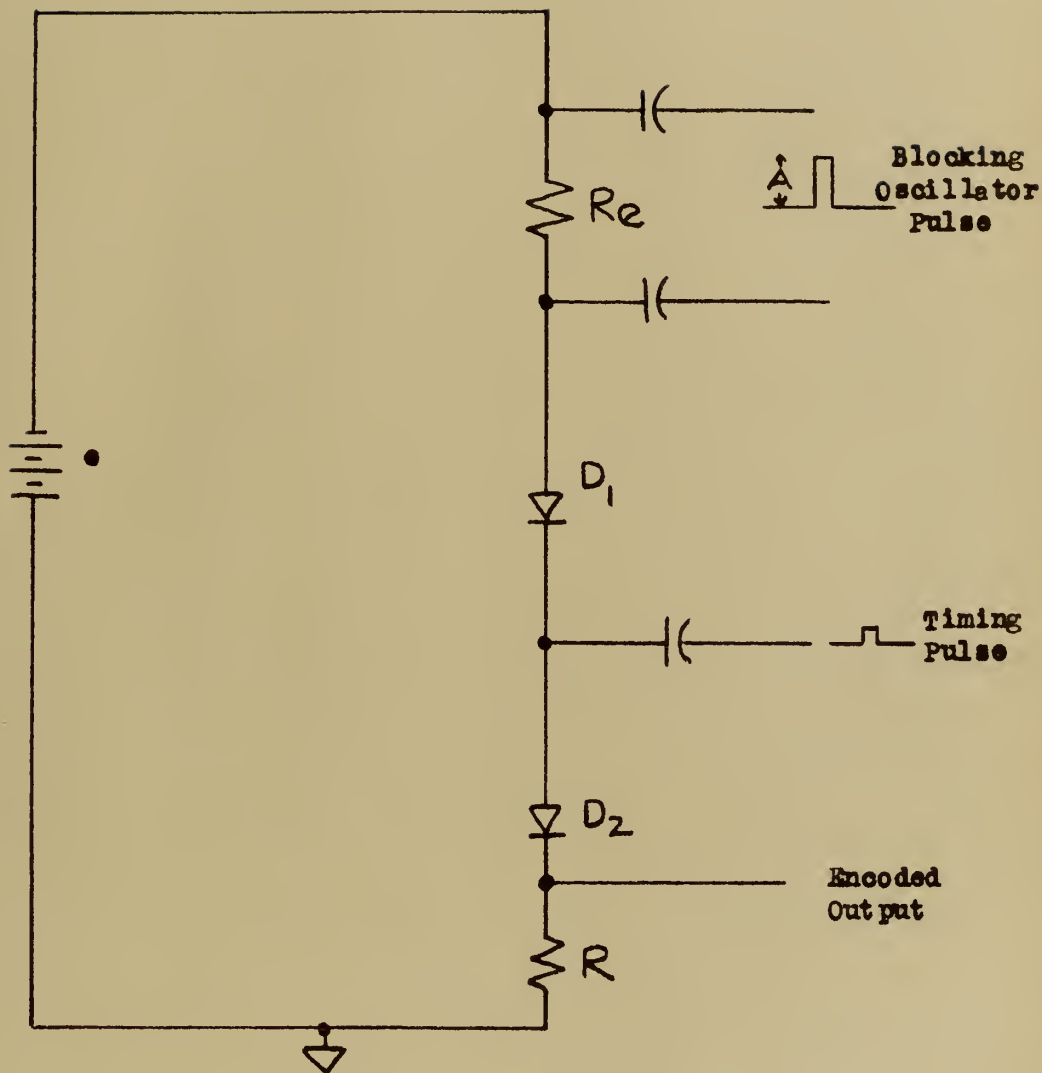


FIGURE 13. Equivalent Circuit of Pulse Frequency Synthesis  
Diode Circuit



to the slope of the signal input. The frequency of the blocking oscillator, or more specifically its off-time is a function of  $C_2$  and the plate to cathode resistance of  $V_2$ . Since the plate to cathode resistance of  $V_2$  is controlled by its grid voltage, it is seen that the frequency of the blocking oscillator is proportional to the rate of change of the signal input. The maximum frequency of the blocking oscillator occurs when  $V_2$  is drawing maximum current (least plate to cathode resistance) due to most positive grid swing, while minimum frequency (longest  $V_1$  off-time) occurs when  $V_2$  is cut off by negative grid swing.

The blocking oscillator feeds the diode circuit  $D_1$ ,  $D_2$  and  $R_4$ . Referring to the equivalent circuit shown in Figure 13, it can be seen that proper operation demands that the blocking oscillator output pulse amplitude be just less than the biasing voltage  $e$ . Then when the small amplitude timing pulse coincides (or nearly so) in time with the blocking oscillator pulse, both diodes are closed (conducting) and an output pulse of approximately  $A$  amplitude results. On the other hand, when the blocking oscillator pulse occurs at a time other than in coincidence with the timing pulse,  $D_1$  and  $D_2$  are open and no pulse appears at the output. All small amplitude timing pulses do appear in the output, however, and it becomes necessary to amplitude discriminate against the unwanted timing pulses.

The pulse frequency synthesis method is remarkably simple compared to the feedback encoder, and has additional advantages in that recovery from saturating input signals, either in frequency or amplitude, is essentially instantaneous.



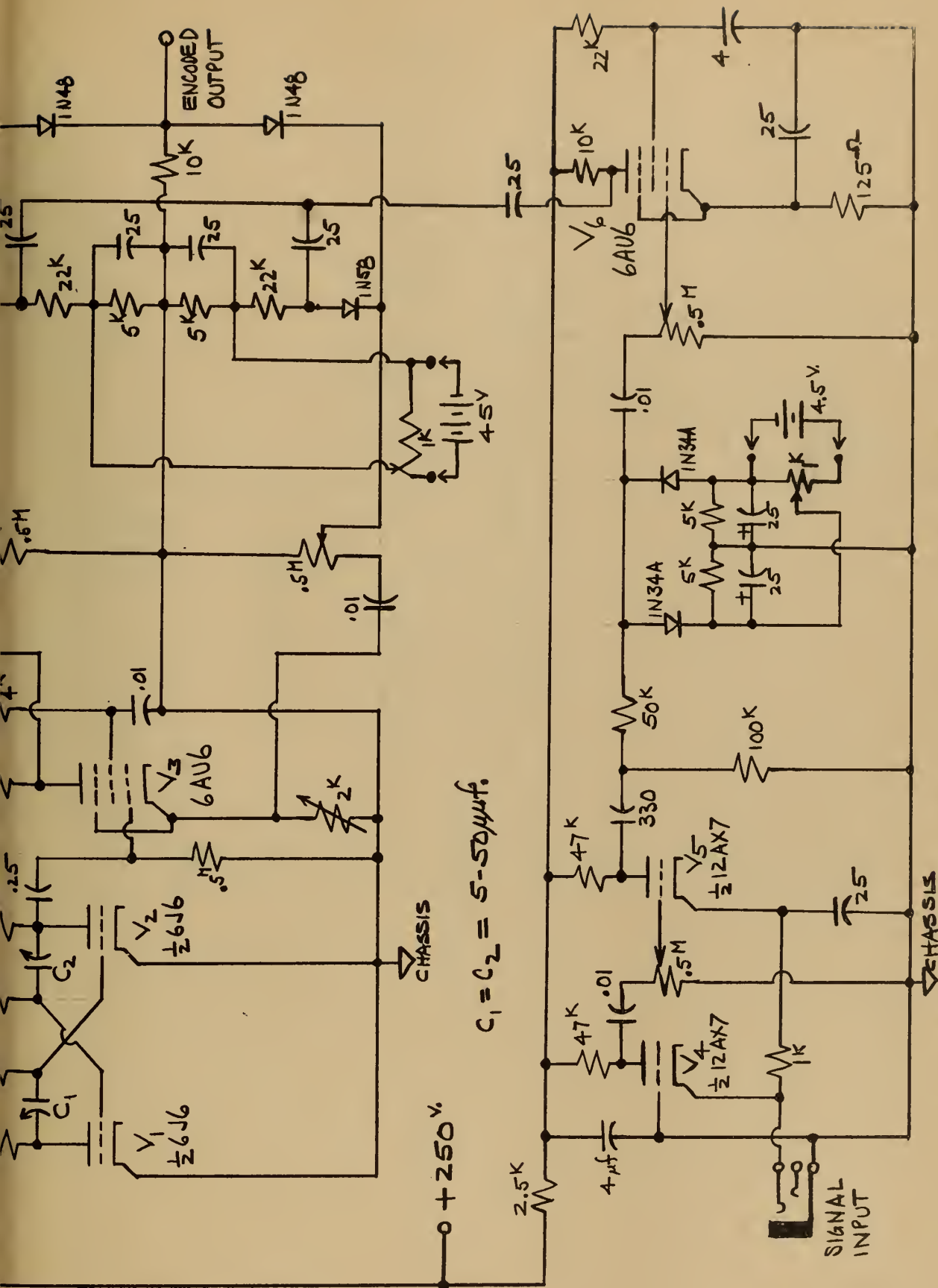
Considerable experimentation was conducted on the gating encoder, block diagram of which is shown in Figure 7. The schematic diagram of the circuit employed in the encoder is shown in Figure 14. The decoder circuit used is shown in Figure 15. The decoder can be seen to consist of a video amplifier followed by an integrator circuit consisting of  $R_1 - C_1$ . This circuit functioned experimentally in the manner predicted, and no further decoding analysis or experimentation was carried out.

Referring to the encoder schematic in Figure 14,  $V_1$  and  $V_2$  constitute a 100 kcps multivibrator with grids returned to B+ for best shaping of the 1.25  $\mu$ second output pulses.  $V_3$  is a paraphase amplifier employed in order to minimize loading effects on the pulse generator. Provision for equalizing the positive and negative pulse amplitudes by the 2 K potentiometer in the cathode circuit of  $V_3$  is to be noted. Pulse amplitudes of about 18 volts appear at the outputs of  $V_3$ .

The signal input is introduced in the cathode of  $V_4$  and  $V_5$  by a carbon button microphone, thus avoiding the requirement for an input transformer. The 500 K potentiometer in the grid circuit of  $V_5$  provides for gain adjustment. The plate output of  $V_5$  is differentiated by the 330  $\mu\mu$ f capacitor and the 100 K resistor and the output of this circuit is fed to the clipper circuit  $D_5-D_6$ . The clipping level is set by adjustment of the 1 K potentiometer and was adjusted for proper operation to give about 30 db of clipping action. The output of the clipper is then coupled to the grid of the gating voltage amplifier  $V_6$ . This voltage is a square wave of amplitude about 0.5 volts.  $V_6$  has an amplification factor of about 50 in this application, and its output is an approximate











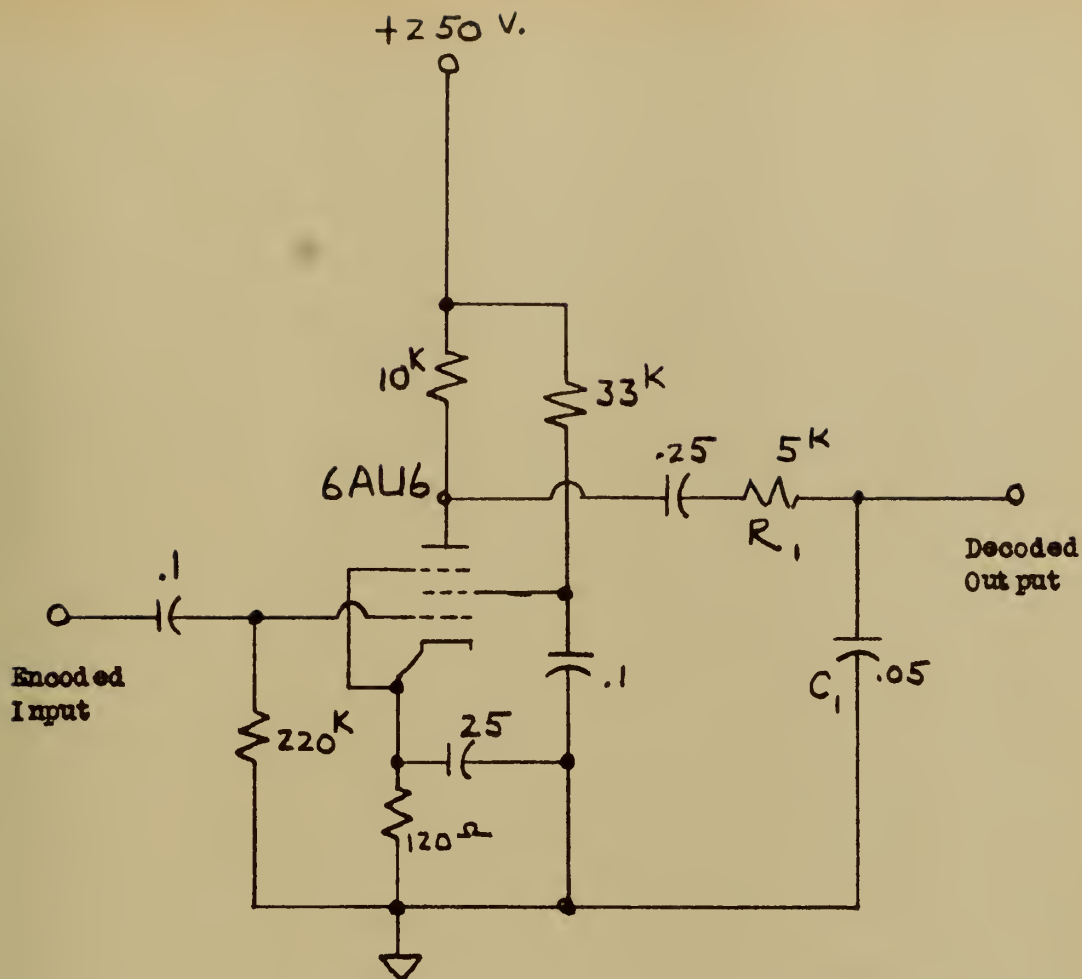


FIGURE 15. Decoder Circuit with Single Integrator,  $R_1$ - $C_1$ .



square wave of amplitude about 25 volts. The gating voltage waveform at the grid of  $V_6$  is excellent, however due to phase distortion in the low frequency gain characteristics of  $V_6$  the gating voltage output waveform is not square, but sufficiently near thereto for purposes of the encoder. This circuit is made up entirely of diodes-- $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$ . Coincidence gates employing vacuum tubes was considered but discarded in favor of the simpler germanium diode circuitry shown. Some of the problems that appear due to finite back resistance and significant forward resistance could be further minimized by use of tube circuits.

The functioning of this encoder can best be seen by reference to the equivalent circuit shown in Figure 16. The amplitude of the gating voltage input is made just equal to  $e$ , both of which are larger than the amplitude of the pulse inputs. Therefore,  $D_1$  does not conduct and in so doing permits positive pulses to the output during the positive portion of the gating cycle. Meanwhile  $D_2$  is conducting and so prevents negative pulses appearing in the output. When the gating voltage is negative,  $D_1$  conducts and denies positive pulses access to the output, while  $D_2$  is non-conducting and negative pulses appear in the output.  $D_3$  and  $D_4$  provide isolation of the encoding circuits while being mixed in the output. Resistances shown as  $R$  in Figure 16 are not employed in the actual circuit, but simulate the output resistances of the paraphase amplifier.

The amplitude distortion of the frequency components was investigated and results are shown in Figure 17. Good agreement with theoretical distortion prediction appears to exist. Several speech intelligibility



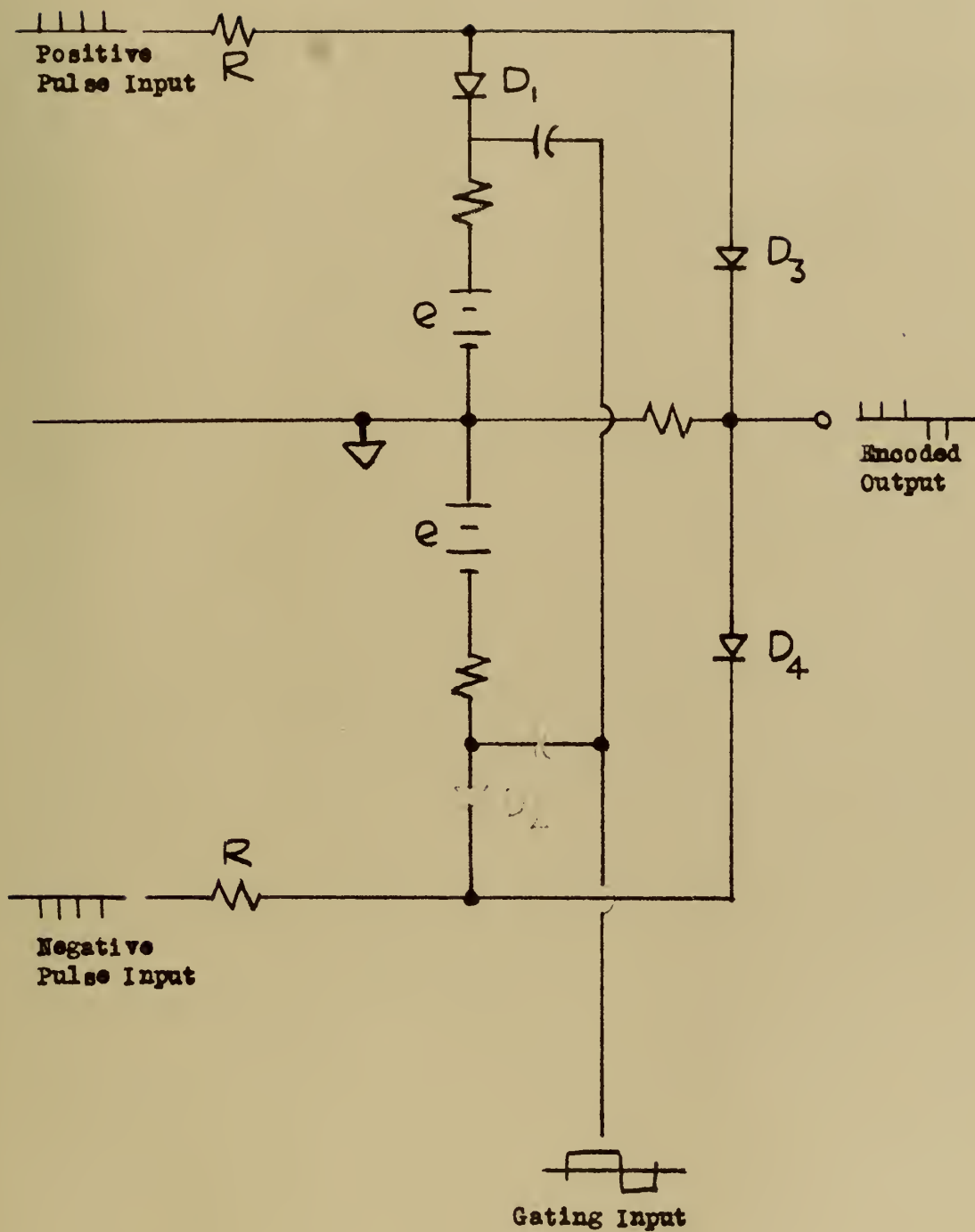


FIGURE 16. Equivalent Circuit of Diode Encoder



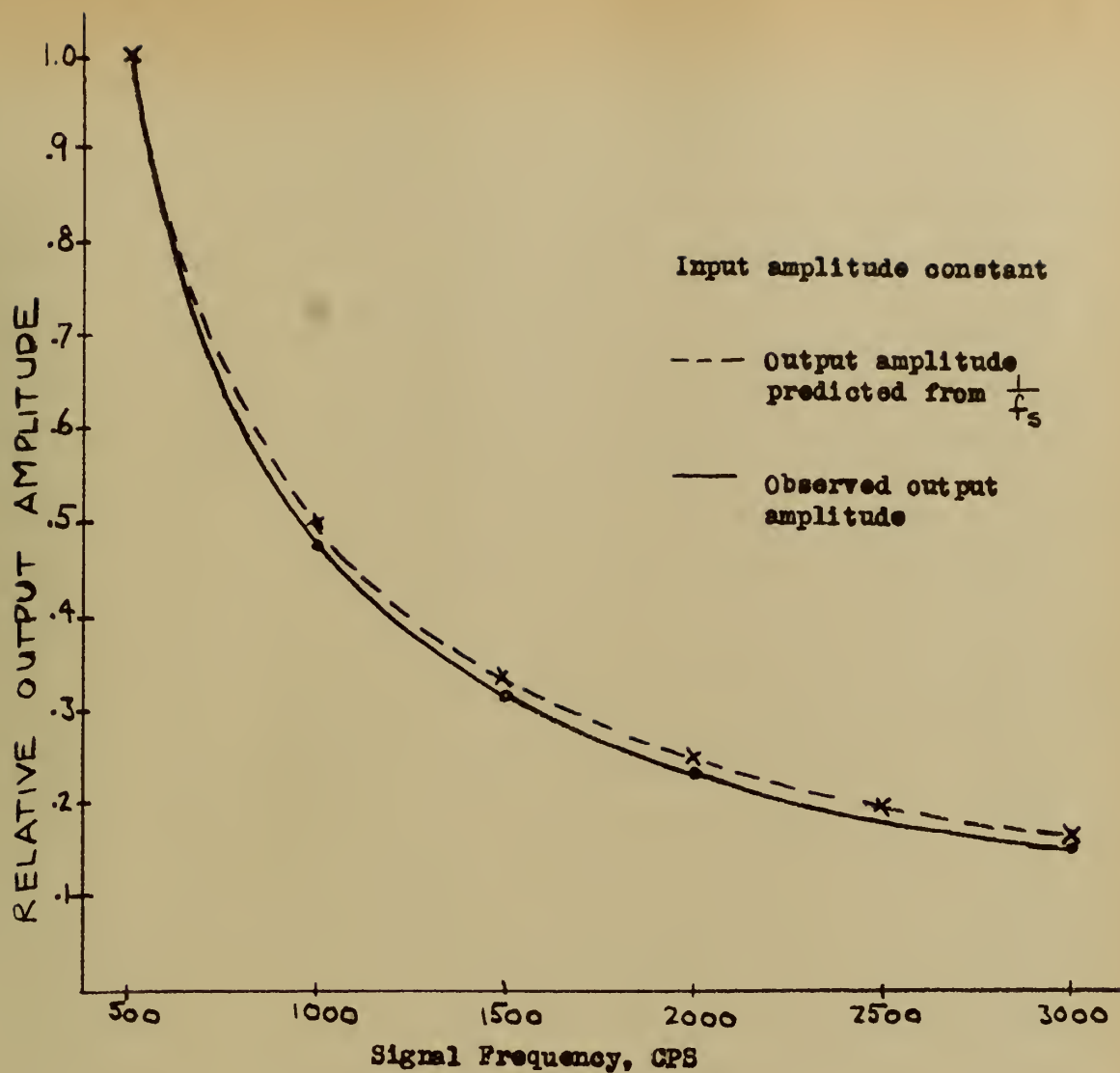


FIGURE 17. Amplitude Distortion of Signal Frequency Components.





tests were made, and it was found that the output was perfectly intelligible, despite loss of certain speech qualities. No effort toward compensation to increase the high frequency gain seemed to be necessary. Further experiment with the system demanded low-pass filters.

The condition of no signal input gives rise to some noise, which is generated by the encoder. This noise was not observed to be of bothersome amplitude at any time during the experiments, however it could be eliminated entirely by use of a no-signal multivibrator which is operative only during conditions of no signal. Block diagram of such a system is shown in Figure 18.

A basically simpler encoder which avoids the no-signal noise problem is pictured in Figure 19. The gated clippers can be made up of diode gates in similar manner to the circuitry shown in the encoder of Figure 14. It is apparent from Figure 19 that the no-signal condition renders both clippers ineffectual and the output pulse of the delay-line generator appears at the output. This method of encoding has disadvantages in that from a multiplexing standpoint, the channel capacity is cut in half. In a single channel system, of course, no disadvantage exists and the arrangement shown in Figure 19 is considered most desirable.



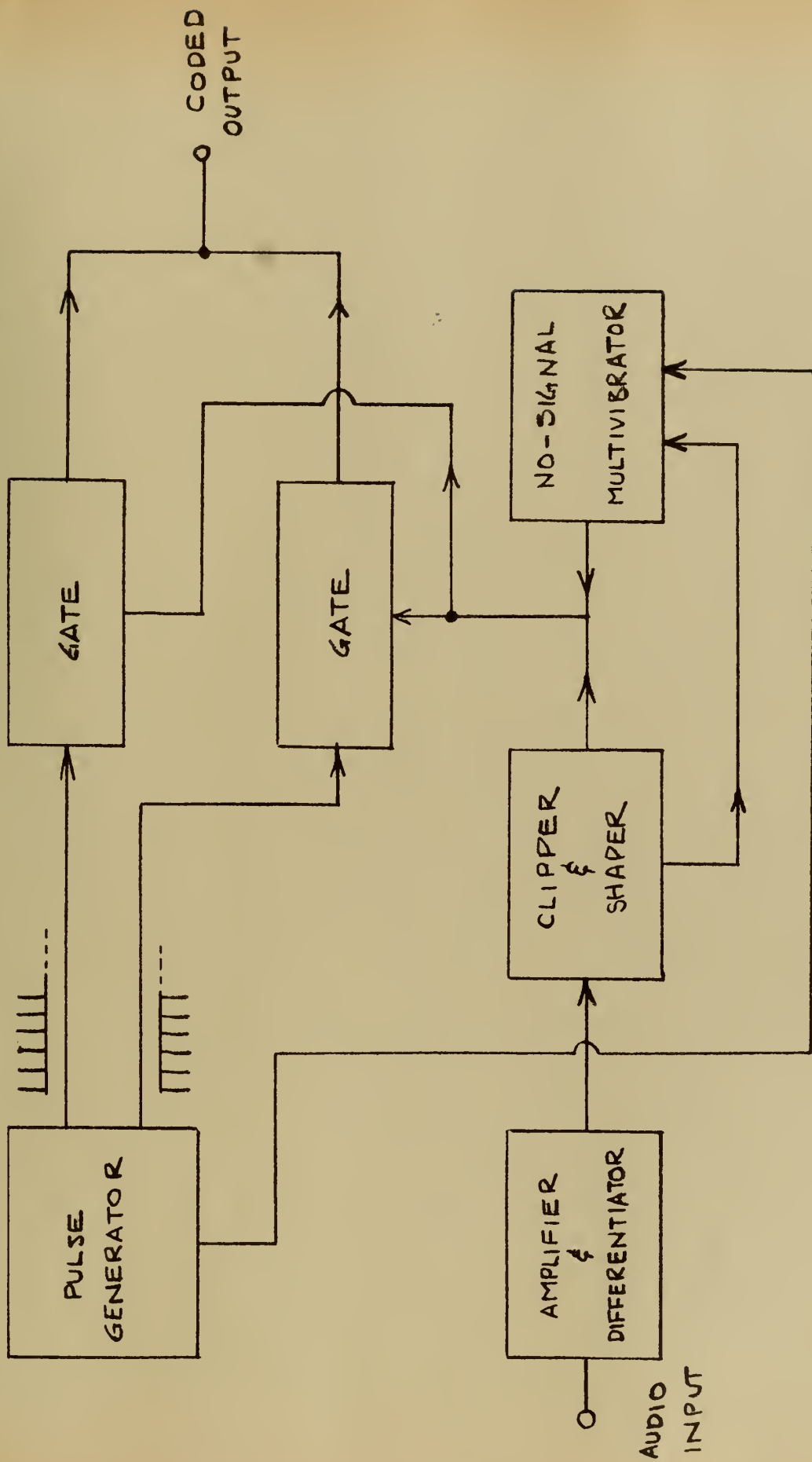


FIGURE 18. GATING ENCODER WITH NO-SIGNAL STABILIZING MULTIVIBRATOR



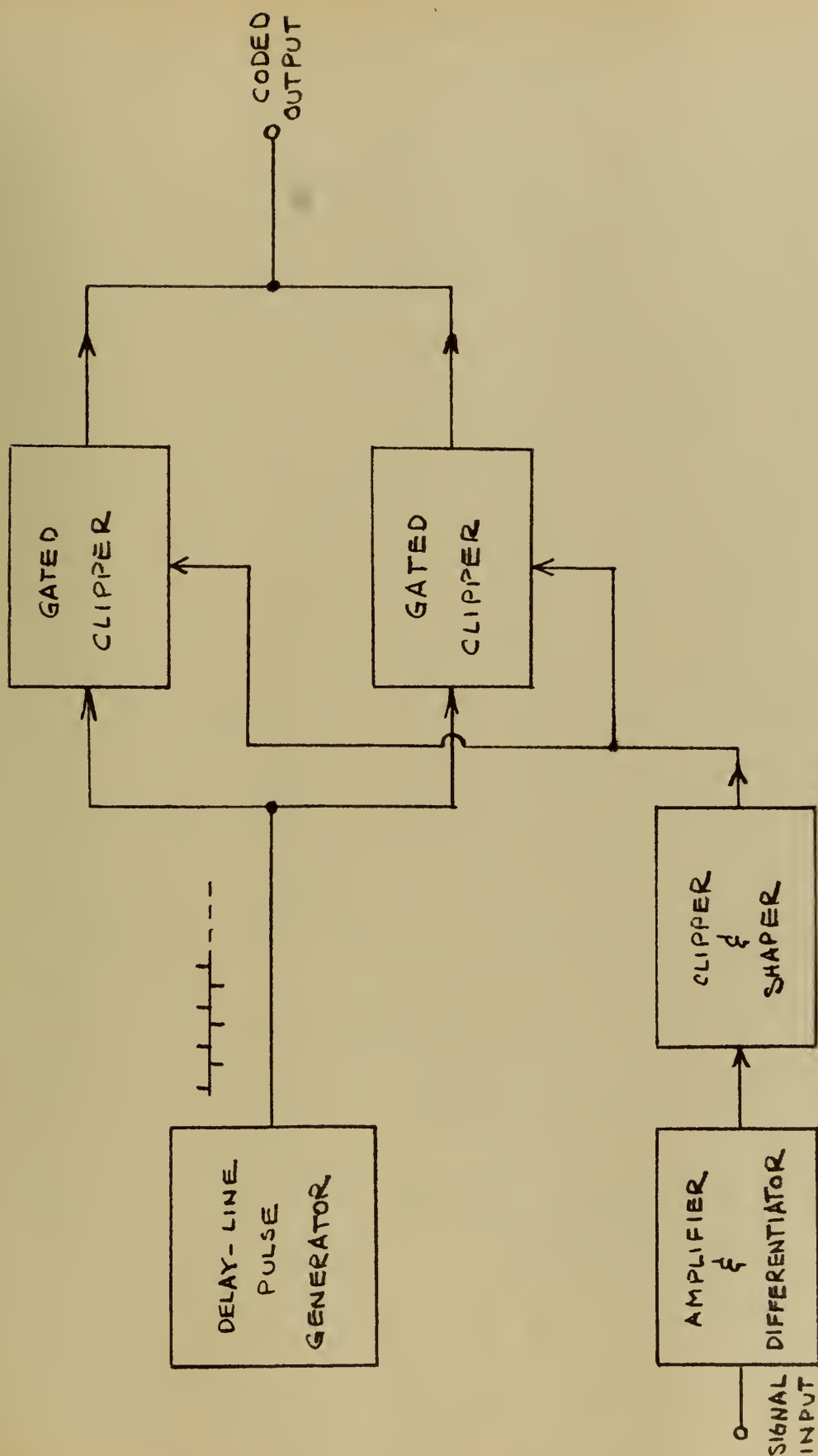


FIGURE 19. GATING ENCODER WITH INHERENT NO-SIGNAL STABILITY



## CHAPTER V

### CONCLUSIONS

It has been pointed out that delta modulation systems have many of the signal-to-noise and regenerative advantages of conventional PCM systems without the disadvantage of circuit complexity which attends conventional PCM. This is an important military advantage because of the high noise level communication channels often encountered. Delta modulation approximates a PCM system from which all amplitude redundancy has been removed. Thus cumulative error through lost pulse information is admitted by the delta modulation system but not by the conventional PCM system. Delta modulation quantizing noise limitations demand high sampling rates (about 100 kops) thus restricting multiplex applications.

Delta modulation is not a rigorously defined technique, but is the term applied to modulation techniques in which the intelligence signal rate of change is used in the modulation process rather than the amplitude of the intelligence signal.

Three methods of generating delta modulation appear feasible as of this writing:

1. Feedback encoders
2. Pulse frequency synthesis encoders
3. Gating encoders

Feedback and pulse frequency synthesis encoders have inherent distortion only in quantizing action. The gating encoders however inherently have both quantizing distortion and a diagonal type of distortion.





All techniques give perfectly intelligible speech transmission. Pulse frequency synthesis and gating encoders are much simpler and give better performance than feedback encoders. The gating encoder transmits less information about the intelligence signal than either of the other techniques.

The gating method of delta modulation generation refers to the technique of transmitting only information as to the polarity of the input signal rate of change, and specifically excludes systems which transmit information as to the magnitude of the input signal rate of change. Therefore the gating method does not refer to a specific circuit but implies only that derivative polarity information is to be transmitted. This method of encoding is consistent with the interpretation of delta modulation as a uni-digit pulse code modulation system.

The circuitry used in the gating encoder may take a variety of forms. In the types discussed in this paper, simplicity of circuits has been achieved through substitution of diode coincidence circuits for those employing vacuum tubes. This simplicity has been bought at the price of a certain degree of waveform deformation due to the magnitudes of the diode forward and reverse resistances. It was found experimentally that this pulse deformation could be tolerated since for decoding purposes, the essential pulse characteristic is that all pulses in the sequence be identical. Thus relatively large rise and fall times could easily be tolerated.

Transmission of delta modulation encoded signals over a radio link presents no special problems. Polar pulses are not required to be



transmitted, so that the problem is only to key the carrier (or sub-carrier) in accordance with unipolarity deltamodulation information.

Deltamodulation systems of some form appear destined to find both military and commercial applications in the future.



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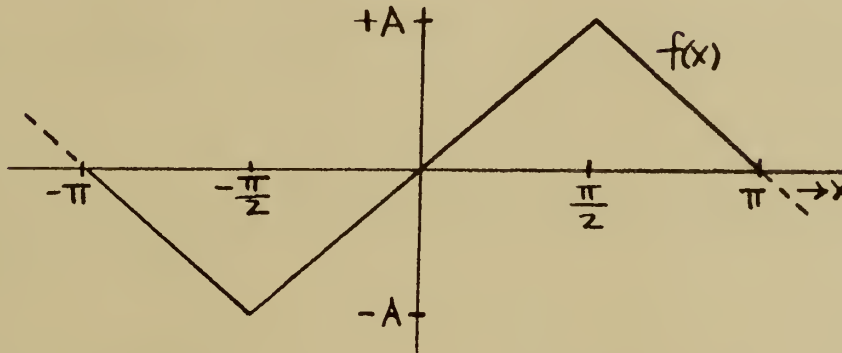
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# APPENDIX 1

## WAVEFORM ANALYSIS

The output waveform of the gating encoder system is approximated by a triangular wave of the following form:



The waveform may be expanded in a Fourier series by determining the coefficients in

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

and for the assumed waveform,  $a_0 = 0$ ,  $a_n = 0$  and no even harmonic terms will exist, so the problem is solved by determining the  $b_n$  for  $n = 1, 3, 5, 7, \dots$

When  $-\pi < x < -\frac{\pi}{2}$  ,  $f(x) = -\frac{2A}{\pi} (\pi + x)$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$  ,  $f(x) = \frac{2A}{\pi} x$

$\frac{\pi}{2} < x < \pi$  ,  $f(x) = \frac{2A}{\pi} (\pi - x)$





$$\begin{aligned}
 b_n &\equiv \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = -\frac{1}{\pi} \int_{-\pi}^{-\pi/2} \frac{2A}{\pi} (\pi+x) \sin nx \, dx \\
 &\quad + \frac{1}{\pi} \int_{-\pi/2}^{\pi} \frac{2A}{\pi} x \sin nx \, dx + \frac{1}{\pi} \int_{\pi/2}^{\pi} \frac{2A}{\pi} (\pi-x) \sin nx \, dx \\
 b_n &= \frac{8A}{n^2 \pi^2} \sin n \frac{\pi}{2} \qquad n = 1, 3, 5, 7, \dots
 \end{aligned}$$

When used as harmonic amplitudes, only the following portion of this expression is needed:

$$C_n = \frac{8A}{n^2 \pi^2}$$











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